This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

EducT
149.01
440

A CONSTRUCTIVE FORM WORK IN ELEMENTARY GEOMETRY

HAILMANN

C. C. BIRCHARD & COMPANY

Educ T 149.01.440

HARVARD UNIVERSITY

LIBRARY OF THE

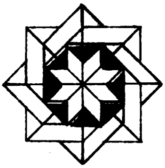
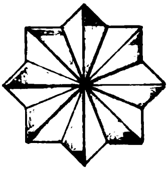
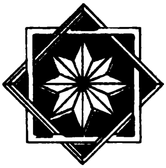
Department of Education

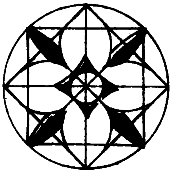
COLLECTION OF TEXT-BOOKS
Contributed by the Publishers

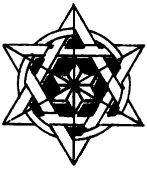
TRANSFERRED
TO
HARVARD COLLEGE
LIBRARY

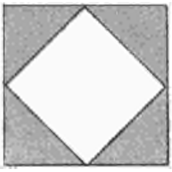
3 2044 097 045 868

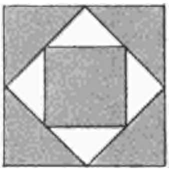
CONSTRUCTIVE FORM WORK

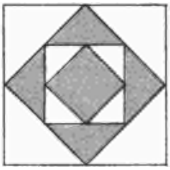




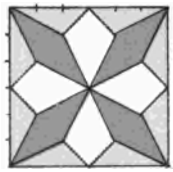


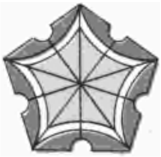


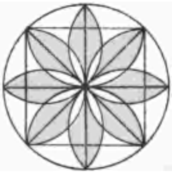


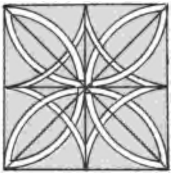


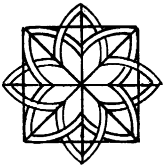


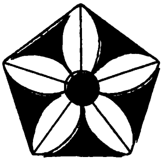


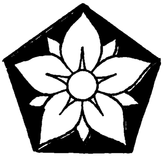


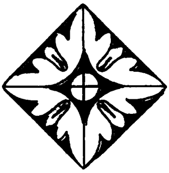


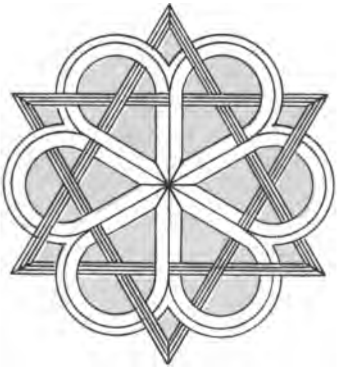


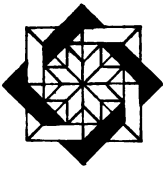
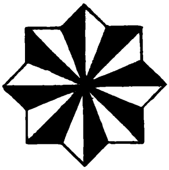
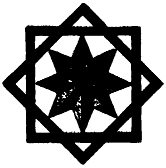
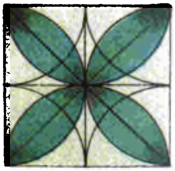


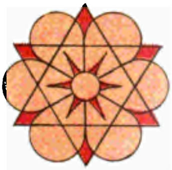
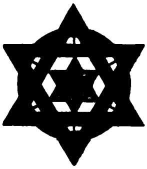


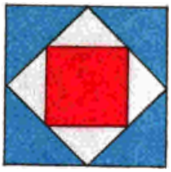


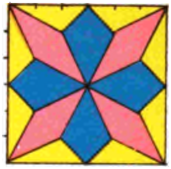


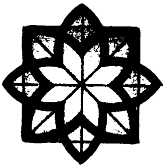


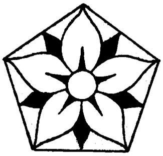





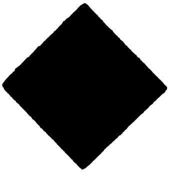













CONSTRUCTIVE FORM WORK

AN INTRODUCTION TO GEOMETRY FOR GRAMMAR GRADES

BY

WILLIAM N. HAILMANN, A.M., PH.D.

BOSTON, MASS.
C. C. BIRCHARD & CO.
1901

~~T 10.4769~~
Educ T 149.01.440
√

April 25, 1903.
Harvard University,
Dept. of Education Library.
Gift of the Publishers.

ED TO
COLLEGE LIBRARY
16 192

COPYRIGHT, 1901,
BY C. C. BIRCHARD & CO.

Norwood Press
J. S. Cushing & Co. — Berwick & Smith
Norwood Mass. U.S.A.

CONTENTS

PART I

PAGE

Circles, Polygons, Angles, Polyhedrons 1

PART II

Composite Curves 40

APPENDIX

Definitions of Geometrical Terms 47

INTRODUCTION.

THE following presentation of rudimentary constructive geometry grew in the schoolroom, and is based on an experience of many years with children between the ages of ten and fourteen.

The purpose, throughout, is to develop clear geometrical notions, to give skill in accurate construction, to cultivate a healthy æsthetic feeling, the power of visualizing creatively in geometrical design, and thus, incidentally, to stimulate genuine vital interest in the study of geometry.

The work, outlined in the problems and exercises, is intended for grammar grades. It may be profitably distributed over the entire grammar school period and accompanied in the upper grammar grades by the study of problems bearing on the equality and similarity of figures and on mensuration, as presented in modern text-books on arithmetic.

The work presupposes on the part of the pupils a certain familiarity with geometrical forms, current in the modern primary school; but it does not presuppose clear definitions and comprehensive classifications of terms, nor the delicacy of touch and accuracy of sight in constructive work which the primary school cannot give and should not aim to give.

The work begins with the study of the circle, since all intelligent geometrical construction rests upon this. From the very start, the knowledge and skill gained are utilized in exercises in accordance with the fundamental purposes of the work as indicated above.

A number of such exercises are suggested. These, although sufficient for the logical march of the work, may be varied and multiplied and extended indefinitely by the teacher. Indeed, the teacher should not hurry through the book. Each problem affords material for many lessons. If the pupil is really to know and to love what he is learning, he must be permitted to linger with it for days and weeks in richly varied exercises that appeal to his analytic power and to his creative fancy.

Under ordinary conditions, the Fifth Grade need not go beyond the first six problems; the Sixth Grade not beyond the first fifteen; the Seventh Grade not beyond Part I. Or, if the work is begun in the upper grammar grades, the task of the Seventh Grade may end profitably with the nineteenth problem.

On the other hand, at no stage should the pupil be restrained, if in his free designs he anticipates the prescribed course.

It will be noticed that, in a number of instances, the pupil is asked to analyze given constructions on the basis of previously acquired knowledge and skill. This practice, too, may, under favorable conditions, be extended profitably beyond the direct requirements of the book.

An appendix has been added in which definitions and classifications are presented in comprehensive form. Care should be taken not to force these upon the pupil's verbal memory. The definitions given, or others equally correct, should be developed with the class through suitable questions. They should rest firmly in the visual and logical memory of the pupils.

Indeed, the pupils should be encouraged to frame definitions from different points of view and to classify terms on the basis of a variety of criteria.

On the other hand, after such exercises the definitions and classifications given in the appendix may be freely used for reference and as convenient conventional formulas in the work of the class.

The tools required for the pupil are a pair of good dividers with a sharp, hard pencil point, a good ruler, and a well-sharpened hard pencil. Whenever feasible, there may be added to these a box of water-colors. Where water-colors are not available, hatching with a hard pencil or shading with a soft pencil can profitably be substituted in emphasizing designs.

The first step in the presentation of the work is dictation by the teacher. For this purpose a few pupils should be called to the blackboard, while the remainder follow the dictation in their seats.

The tools for blackboard work are a good pair of blackboard compasses, a ruler, and a well-sharpened crayon. Instead of the compasses, the pupils at the blackboard may use a string to one end of which a well-sharpened crayon is tied; and instead of a ruler, they may use a chalked string to be manipulated like the carpenter's chalk-line.

PART I.

CIRCLES, POLYGONS, ANGLES, AND POLYHEDRONS.

Problem 1.—*How to draw a circle* (Fig. 1).

Select a point near the middle of your paper thus, **x**. Mark it *o*. Open your dividers to the distance of two inches. Place the metal point of the dividers in the point *o*, and with the pencil point draw a curved line all around the point *o*.

The space inclosed within the curved line is a *circle*.

The curved line is the *circumference* of the circle.

The point *o* is the *centre* of the circle.

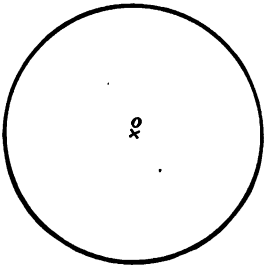


Fig. 1.

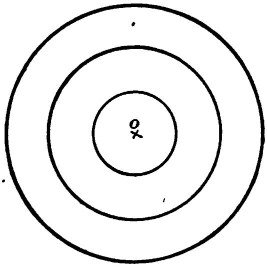


Fig. 2.

EXERCISES.

1. Paint the circle of any color that may please you.
2. Draw three circumferences from the centre (Fig. 2).

(These three circles are said to be *concentric*, i.e. having the same centre.)

3. Paint the inner circle red; the inner ring, yellow; the outer ring, blue.

PROBLEM 2.—*How to draw the diameter of a circle* (Fig. 3).

Draw a circle as in Problem 1. From left to right, draw through the *circle* and through its *centre* a straight line. Name the points in which the straight line meets the *circumference*, respectively, *a* and *b*.

This straight line is a *diameter* of the circle.

The diameter divides the circle into two equal parts or *semicircles*. (*Semi* means half.)

The diameter also divides the circumference into two equal parts or *semi-circumferences*.

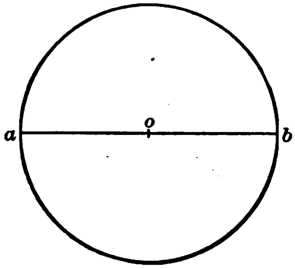


FIG. 3.

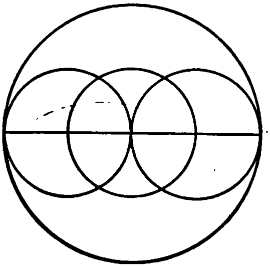


FIG. 4.

EXERCISES.

1. Paint the two semicircles in two different colors (orange and blue, yellow and violet, or red and green).

2. Draw Figure 4 and paint the parts as may please your taste.

3. Draw Figure 5 and paint as you please.

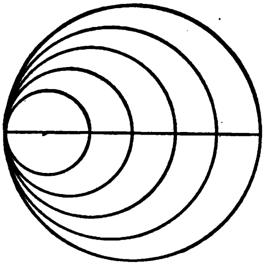


FIG. 5.

PROBLEM 3.—*How to divide a circle into four equal parts* (Fig. 6).

Draw the circumference of a circle and its diameter as in Problem 2. Open the dividers a little wider. Then, with the point *a* as a centre, draw a small curve above the middle of the upper semi-circumference; and, with the point *b* as a centre, draw another curve across the first. Name the point of intersection *n*. Lay the edge of the ruler over the point *n* and the centre of the circle, and draw a second diameter of the circle. Name the second diameter *cd*.

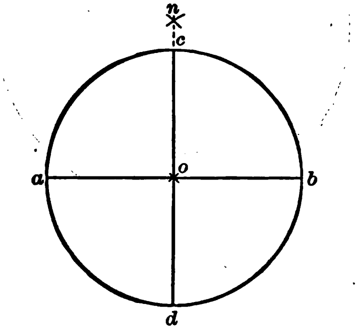


FIG. 6.

The two diameters divide the circle into four equal parts or *sectors*. Each sector is called a *quadrant* (one-fourth). The two diameters also divide the circumference into four equal parts or *arcs*, and each arc is called a *quadrant*. The two diameters make four equal *angles* at the centre of the circle. The diameters, therefore, are said to be *perpendicular* to each other.

The quadrant is said to be the measure of the right angle. The two diameters *bisect* each other, *i.e.* they cut each other into two equal parts.

Each half measures the distance between the centre and the circumference. This half is called a *radius* of the circle. The four *radii* of the circle are equal.

EXERCISES.

1. Paint the sectors, using two colors and giving to opposite sectors the same color.

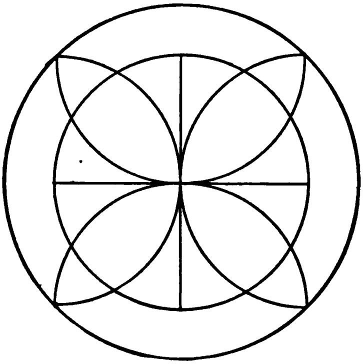


FIG. 7.

2. Study Figure 7; tell how it was drawn; draw a similar design; color it to please your taste.

3. Draw a design of your own, starting with Figure 6.

Degrees and Angles. — For purposes of measurement, the circumference of the circle is divided into 360 equal parts.

Each part is called a *degree.* Its symbol is a small ring (°) placed to the right and above the figure (360°).

Degrees measure arcs, angles, and sectors.

How many degrees in a quadrant? in a semi-circumference? in a right angle? in two right angles?

The two straight lines that form an angle are called its *sides.*

The point where the two sides of an angle meet is called its *vertex* (plural, *vertices*).

PROBLEM 4. — *How to inscribe a square in a circle* (Fig. 8).

Draw the circumference of a circle with a radius of two inches; and in the circle draw two perpendicular diameters, as in Figure 6. Name the horizontal diameter *ab*, and the vertical diameter *cd*. Draw straight lines connecting *a* with *d*, *d* with *b*, *b* with *c*, and *c* with *a*.

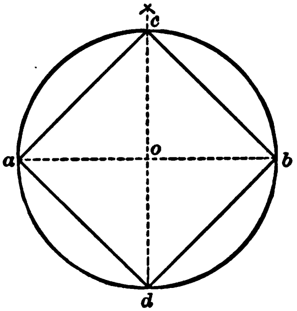


FIG. 8.

The figure *acbd* is a *square*. The vertices of its angles lie in the circumference of the circle. It is, therefore, said to be *inscribed* in the circle.

The straight lines, *ad*, *db*, *bc*, and *ca*, considered separately, are *chords* of the circle.

A chord divides the circle into two parts. Each part is called a *segment* of the circle.

EXERCISES.

1. Carefully cut out the circle (with a knife or a pair of scissors). Then cut off the outer *segments* formed by the *chords* (*ad*, *db*, *bc*, *ca*). What figure have you left? Place the four segments so as to restore the original circle. Then drop the square in its original place.

2. Draw again the circumference of a circle and inscribe a square. Then paint the segments of one color and the inscribed square of another.

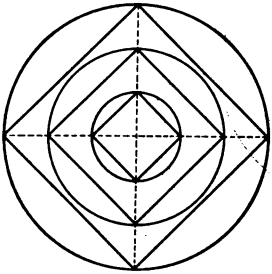


Fig. 9.

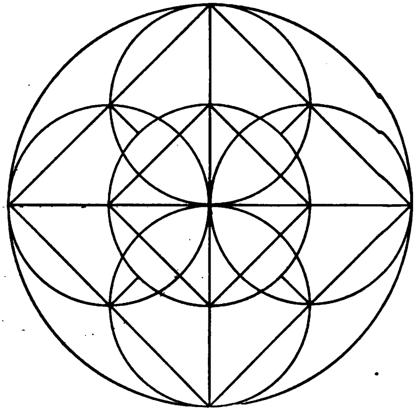


Fig. 10.

3. Study Figure 9. Tell (orally or in writing) how it was drawn. Draw a similar design and color it so as to

show clearly in each circle the outer segments and inscribed squares.

4. Draw a design of your own, starting with Figure 8.
5. Study Figure 10, and tell how it was drawn.

PROBLEM 5. — *How to divide a circle into eight equal sectors* (Fig. 11).

Draw the circumference of a circle and divide it into four quadrants. Name the diameters *ab* and *cd*. With a suitable radius and the points *c* and *b* successively as centres, determine the point *m*; then, with the same radius and points *b* and *d* successively as centres, determine the point *n*. Through the points *m* and *n* draw the diameters *ef* and *gh*.

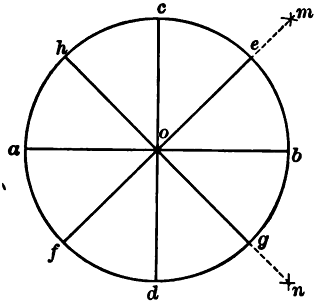


FIG. 11.

The new diameters *bisect* the quadrants.

The new diameters also *bisect* the right angles of the quadrants.

The four diameters divide the circumference into eight equal arcs, and the circle into eight equal sectors, called *octants*.

How many degrees measure the arc of an octant?

How many degrees measure each angle at the centre of the circle?

An angle, smaller than a right angle, is said to be *acute* (or sharp).

An angle, larger than a right angle, is said to be *obtuse* (or blunt).

What radii make acute angles with the radius *ao*?

What radii make obtuse angles with the radius *ao*?

EXERCISES.

1. Paint the sectors of the figure in two colors, giving the same color to *opposite* sectors and different colors to *adjacent* sectors.

2. Draw again the circumference of a circle and divide it into octants. Connect the outer points of the radii of each octant by a chord (Fig. 12). The eight-sided figure you have drawn is called an *octagon*. Paint the octagon in any one color, and the outer segment of the circle in another color.

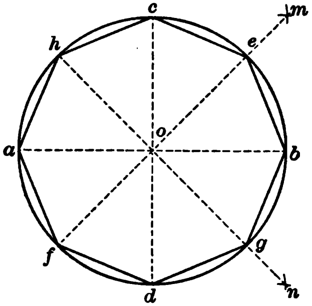


FIG. 12.

The octagon is said to be *inscribed* in the circle. Why?

The sides of the inscribed octagon are equal. They form, together, the *perimeter* of the octagon.

The angles at the perimeter *ahc*, *hce*, etc., are obtuse angles.

These angles, too, are equal.

Because its sides and angles are respectively equal, the octagon is said to be *regular*.

3. Draw Figure 13, and color it so as to show distinctly one of the inscribed squares and the corners of the other inscribed square.

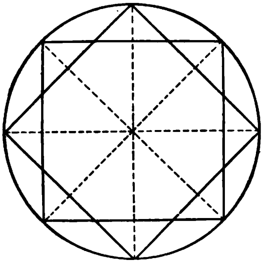


Fig. 13.

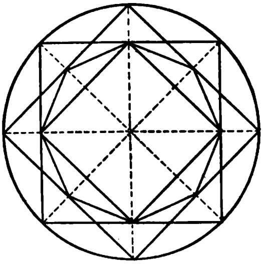


Fig. 14.

4. Draw Figure 14, and color it so as to bring out the central square, the octagon, and the remaining portions of the two squares inscribed in the circle.

5. Study Figure 15; determine what line used in its construction has been erased; construct the figure and color it to your taste.

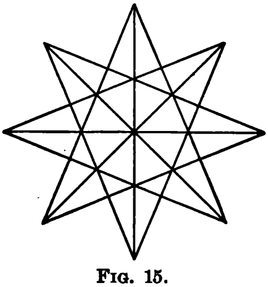


Fig. 15.

PROBLEM 6.—*How to divide a circle into six equal sectors* (Fig. 16).

Draw the circumference of a circle with a radius of two inches. Draw the diameter from left to right (the horizontal diameter) and name it *ab*. With the same radius and *b* as a centre, draw short arcs across the circumference above and below the diameter. Name the points, thus determined, *d* and *e*. Similarly, with *a* as a centre, determine the points *f* and *c*. Draw the diameters *dc* and *ef*.

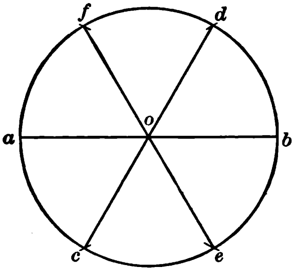


FIG. 16.

The three diameters divide the circle into six equal sectors, called *sextants*.

They also divide the circumference into six equal arcs. These, too, are called *sextants*.

How many degrees in each arc? How many degrees in each acute angle at the centre of the circle?

EXERCISES.

1. Paint the sectors with the six colors of the rainbow (red, orange, yellow, green, blue, violet).

2. Study Figure 17, draw a similar one, and paint it to your taste.

3. Study Figure 18, draw a similar one, and paint it to your taste.

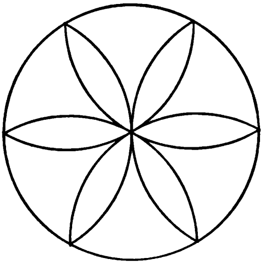


FIG. 17.

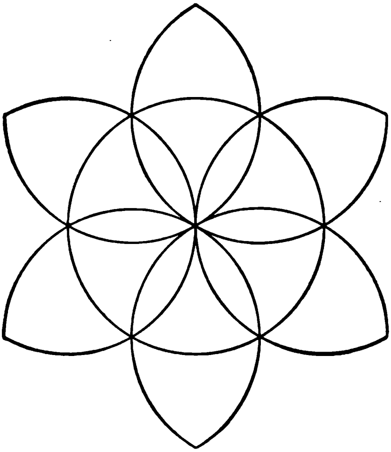


FIG. 18.

4. Invent any similar design, using only arcs of circles, no straight lines.

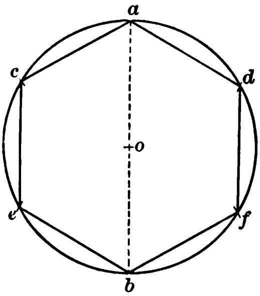


FIG. 19.

5. Study Figure 19, draw one similar to it, and tell in writing how you drew it.

The inscribed figure has six sides; it is called a ***hexagon.***

Because its sides and angles are respectively equal, it is said to be a ***regular*** hexagon.

Because all the vertices of its angles lie in the circumference of the circle, it is said to be ***inscribed.***

6. How do the sides of the inscribed regular hexagon compare in length with the radius of the circle?

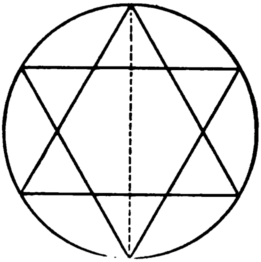


FIG. 20.

7. Study Figure 20, draw one similar to it, and color it so as to show the central hexagon and the points of the star.

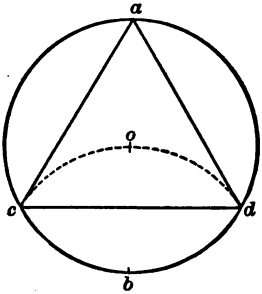


FIG. 21.

8. Study Figure 21 and draw one similar to it.

The regular inscribed figure formed by the three equal chords is an ***equilateral*** (equal-sided) ***triangle.***

How many degrees in each of the three equal arcs?

Paint the equilateral triangle yellow and the three segments blue.

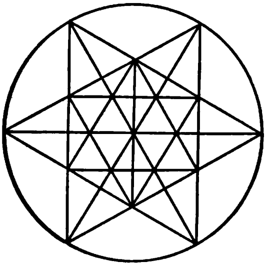


FIG. 22.

9. Study Figure 22 and describe it.
10. Draw this design or invent a similar one and color it to your taste.

PROBLEM 7.—*How to bisect a given straight line* (Fig. 23).

Name the ends of the line, respectively, *a* and *b*. With *a* as centre and with a radius longer than one-half of the line, draw small arcs above and below the middle of the line. With the same radius and *b* as a centre, draw small arcs across the first two. Name the points of intersection,

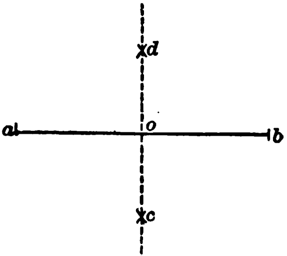


FIG. 23.

respectively, *d* and *c*. Draw a straight line through the points *d* and *c*. This line will bisect the line *ab*. Name the point of intersection of the two straight lines *o*.

EXERCISES.

1. Show, by drawing the circumference of a circle from the proper centre, that *ab* is the diameter of a circle and that *oa* and *ob* are radii of this circle.

2. Show, by suitable construction, that the four angles at the point of intersection of the two straight lines are at the centre of a circle, and that the two straight lines divide this circle into quadrants.

How many degrees in each of these angles?

What kind of angles are they?

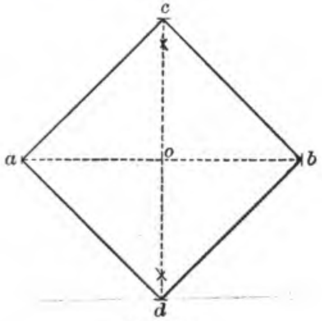


FIG. 24.

3. Study the square of Figure 24. Tell how it was drawn. Draw two squares similar to it, touching at the point *b*.

The lines *ab* and *cd* are the diagonals of the square.

The diagonals divide the square into four equal triangles.

The diagonals bisect the angles of the square.

The two diagonals are *perpendicular* to each other.

How many degrees in each angle of the four triangles?

4. Draw a simple design, starting with Figure 24, using only arcs, and color the design to suit your taste.

5. Draw a simple design, starting with Figure 24, using only straight lines, and color the design to suit your taste.

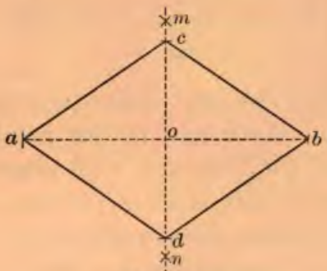


Fig. 25.

6. Study Figure 25. Tell how it was drawn. Draw a figure similar to it.

This four-sided figure (Fig. 25) is called a *rhombus* or *lozenge*.

What effect do the two diagonals have upon the rhombus?

What effect do the diagonals have upon the angles of the rhombus?

What kinds of angles has the rhombus, when you think the diagonals away? How do the opposite angles of the rhombus compare?

7. Draw a simple design, starting with a lozenge, and color it.

8. Draw a lozenge. Then, on each of its sides, draw another lozenge and color the design.

PROBLEM 8.—*How to draw a line perpendicular to a given straight line at any given point* (Fig. 26).

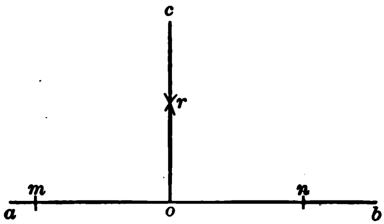


FIG. 26.

Draw a straight line, *ab*. Select any point in this line and mark it *o*. With any radius, not longer than the shorter piece of the line, and *o* as centre, cut off equal parts *om* and *on* on both sides of point *o*. With the same radius and *n* as centre, draw a second arc across the first. Mark the point of intersection *r*. Draw *co*, which is perpendicular to *ab* at the point *o*.

EXERCISES.

1. Show by construction that *mn* is the diameter of a circle, that *o* is the centre of the same circle, and that *co* lies in the direction of the diameter perpendicular to *ab* (see Fig. 23).

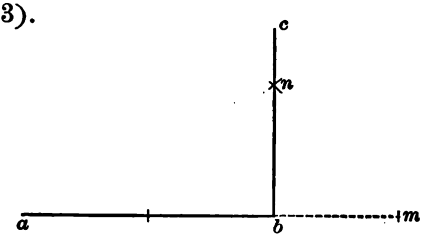


FIG. 27.

2. In Figure 27, the line *ab* was drawn. The problem was set to draw a line perpendicular to *ab* at the point *b*.

Study the construction by which this problem was solved, and make a similar construction.

PROBLEM 9.—*How to draw a square on a given straight line* (Fig. 28).

Draw a straight line *cb*, perpendicular to *ab* at the point *b*. Make this line equal to *ab*. With *ab* as radius and *a* as centre, draw an arc over *a*. With *c* as centre and the same radius, draw a second arc across the first. (Mark the point of intersection *d*.) Draw *ad* and *cd*. *abcd* is the required square.

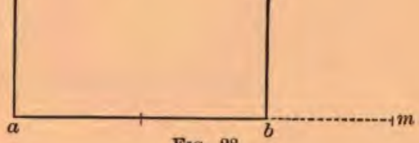


FIG. 28.

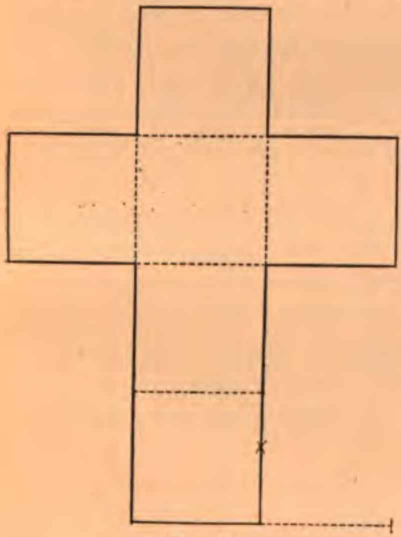


FIG. 29.

EXERCISES.

1. How to construct a cube. Study Figure 29. Construct a similar figure on stiff paper, and after creasing the paper in the dotted lines, construct a cube.

2. Draw an *oblong rectangle* with two given sides (Fig. 30).

How do the opposite sides of the oblong rectangle compare?

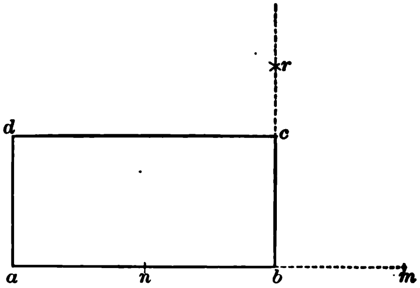


FIG. 30.

What kind of angles in the oblong rectangle?

3. Construct a square prism with the help of Figure 31.

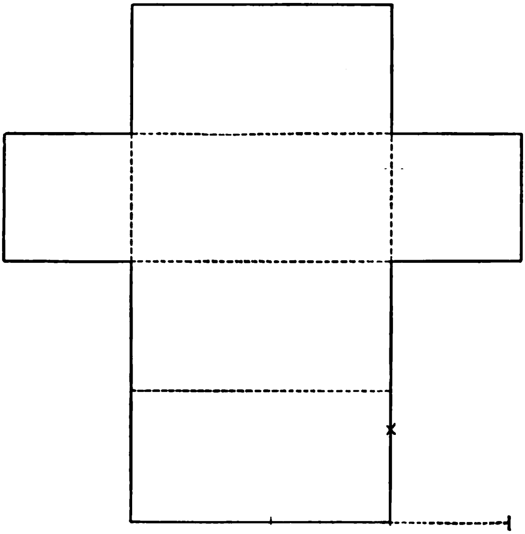


FIG. 31.

Problem 10.—*How to divide a square into four equal, smaller squares* (Fig. 32).

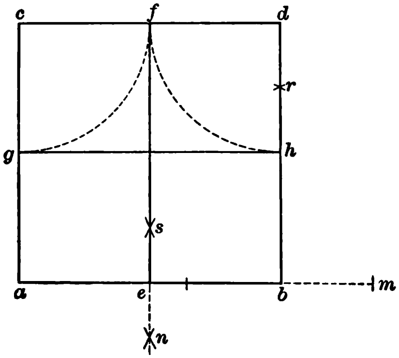


Fig. 32.

Draw a square on a given line, *ab*. Draw through the square a perpendicular to *ab*, at the middle of *ab*. With *cf* as radius and *c* and *d*, successively, as centres, draw the arcs *fg* and *fh*. Draw the straight line *gh*.

The line *ef* bisects the square into two oblong rectangles. It bisects the lines *ab*, *cd*, and *gh*; and it is perpendicular to these three lines.

Similarly, the line *gh* bisects the square, each of the two oblongs *aefc* and *befd*, and the lines *bd*, *ef*, and *ac*, and is perpendicular to these three lines.

The lines *ef* and *gh* are the diameters of the square. They are perpendicular to each other and bisect each other at the centre of the square. They divide the square into four equal, smaller squares.

EXERCISES.

1. Draw Figure 33 and color it with two colors.

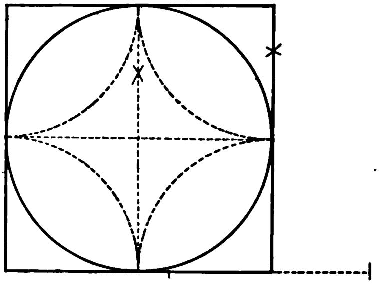


FIG. 33.

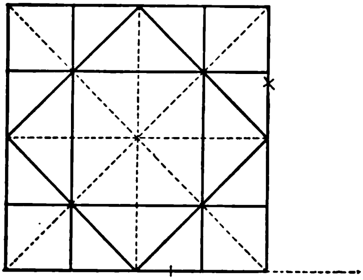


FIG. 34.

2. Draw Figure 34 and color it to your taste.

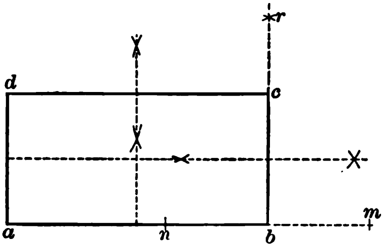


FIG. 35.

3. Draw and color a design on the basis of Figure 33 or 34.

4. Divide a given oblong rectangle into four equal, smaller oblong rectangles and explain your work (Fig. 35).

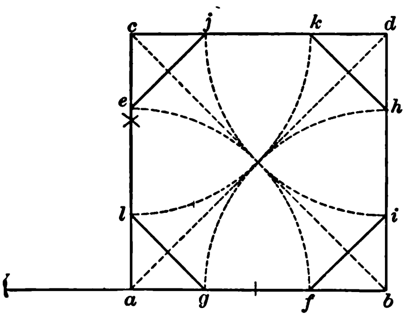


FIG. 36.

PROBLEM 11.—*How to inscribe a regular octagon in a given square* (Fig. 36).

The construction is shown in Figure 36. Study it; explain it; and make the construction, drawing the octagon in heavier lines.

EXERCISES.

1. Study and repeat Figure 37. Color it, so as to show clearly the four wings of the cross.

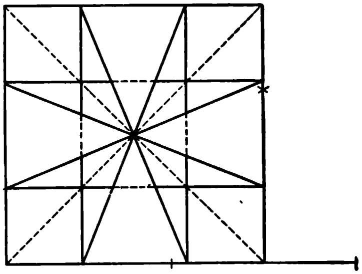


FIG. 37.

2. Study Figure 38. Draw it, and use it for a color design.
3. On the basis of Figure 36, draw a color design.

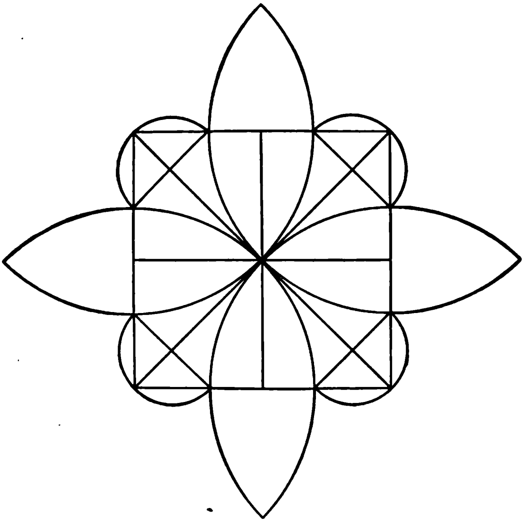


FIG. 38.

PROBLEM 12. — *How to circumscribe a square about a given circle.*

The construction is shown in Figure 39. Study it. Notice that the sides of the circumscribed square are equal to the diameter of the circle. Explain and repeat the construction.

Each side of the circumscribed square touches the circumference in only one point.

On this account, it is said to be a *tangent* (touching line) to the circle.

Each tangent is perpendicular to the diameter of the circle at the point of contact.

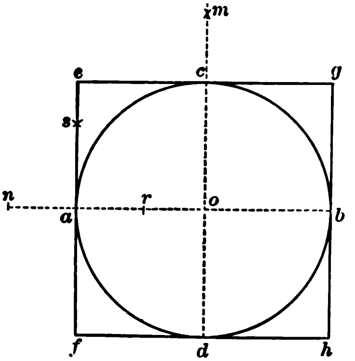


FIG. 39.

EXERCISES.

1. Draw and color a design on the basis of Figure 39.

2. Inscribe a hexagon in a given circle. Then circumscribe a hexagon about the same circle, and use the figure for a suitable design (Fig. 40).

3. Construct an octagonal prism.

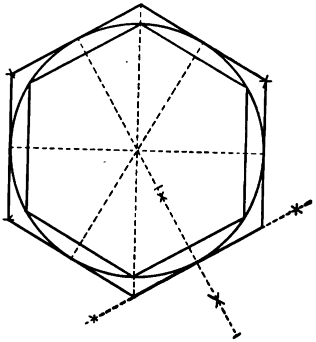


FIG. 40.

PROBLEM 13. — *How to draw an equilateral triangle on a given side* (Fig. 41).

Draw the given side *ab*. With *ab* as radius and *a* and *b* successively as centres, draw small intersecting arcs over the middle of *ab*. Connect the point of intersection *c* with *a* and *b*.

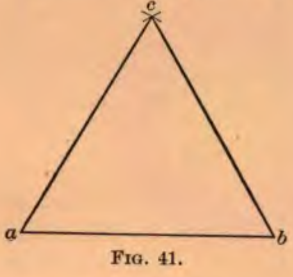


FIG. 41.

EXERCISES.

1. Draw Figure 42 on cardboard or stiff paper. Cut the cardboard or crease the paper in the dotted lines.

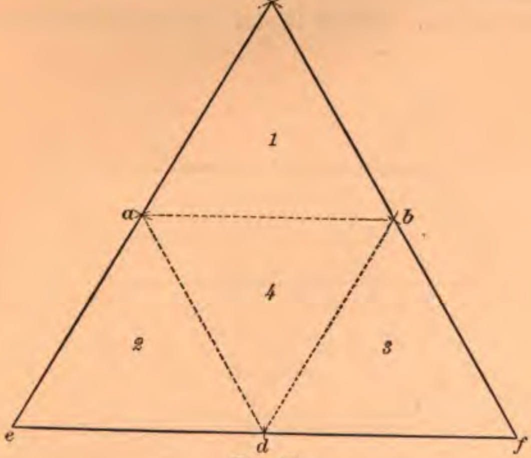


FIG. 42.

Fold the triangles 1, 2, and 3 inward until the points *c*, *e*, and *f* meet. You will have constructed a *tetrahedron* or regular four-sided solid.

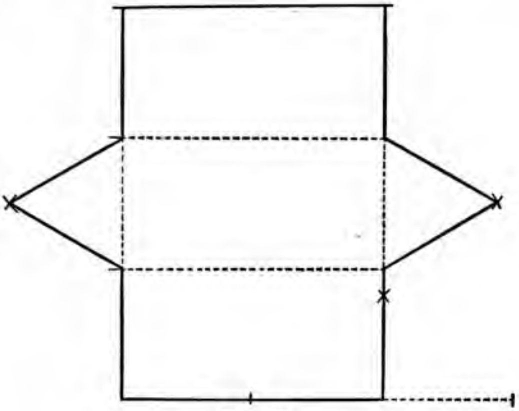


FIG. 43.

2. Figure 43 shows the construction of a three-sided or triangular prism. Study it and construct the prism.

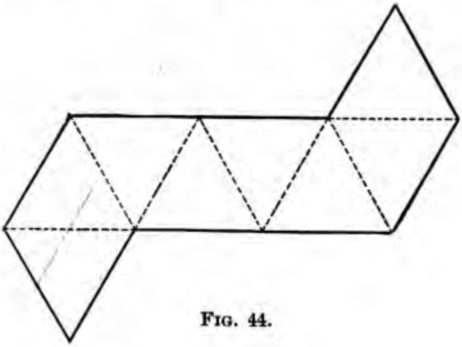


FIG. 44.

3. Figure 44 shows the construction or a regular *octahedron* or eight-sided solid.

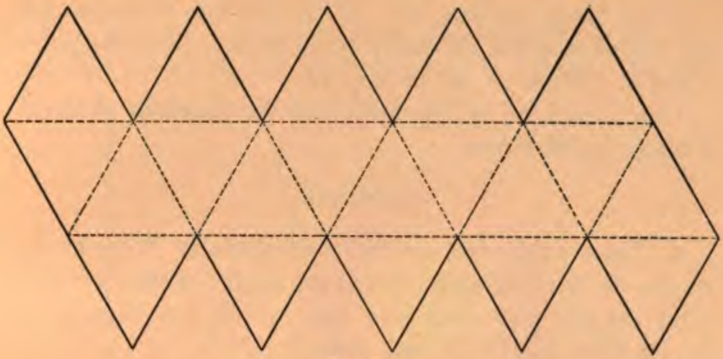


FIG. 45.

4. Figure 45 shows the construction of a regular *icosahedron* or twenty-sided solid.

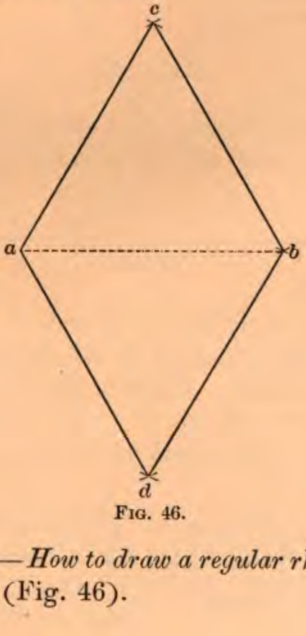


FIG. 46.

PROBLEM 14. — *How to draw a regular rhombus or lozenge with a given side* (Fig. 46).

Draw the given line, *ab*. With this line as radius and the points *a* and *b*, successively, as centres, find the points *c* and *d*. Draw *ac*, *bc*, *ad*, and *bd*.

Into what kind of triangles does the diagonal *ab* divide the regular rhombus?

EXERCISES.

1. Draw a rhombus and its two diagonals. What kind of angles do the diagonals form at the centre of the rhombus? How do the four triangles, into which the rhombus is divided, compare with each other?

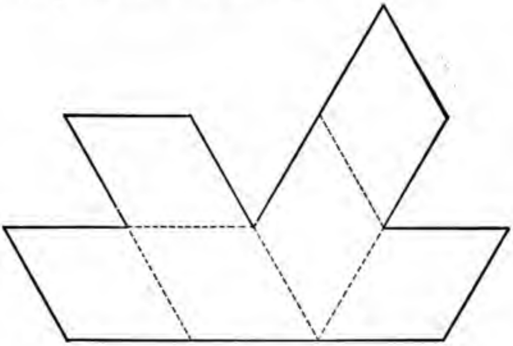


FIG. 47.

2. Draw Figure 47 on cardboard or stiff paper. By folding the lozenges inward in the dotted lines, you will obtain a *rhombohedron*. Describe the rhombohedron.

PROBLEM 15. — *How to draw isosceles triangles with two given sides* (Fig. 48).

The sides *ab* and *bc* are the given sides. They may be of any length. Draw either of the two sides, *ab* or *bc*, as the base of the triangle. Then with the other side as radius and the ends of the base, successively, as centres, find the vertex of the triangle, *a* or *c*. Draw the two

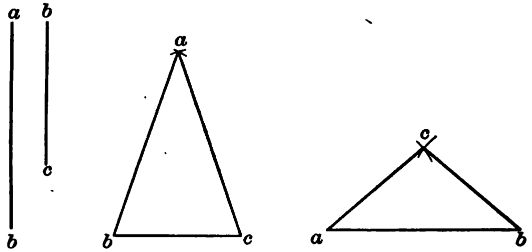


FIG. 48.

equal sides, *ac* and *bc*, or *ba* and *ca*. The resulting triangle is said to be *isosceles*, because it has two sides equal.

EXERCISES.

1. Draw *ad* perpendicular to *bc* (Fig. 49) and *cd* respectively perpendicular to the base of the triangle.

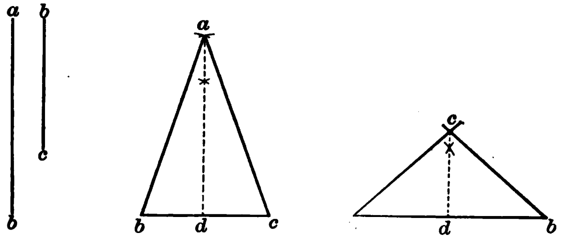


FIG. 49.

In the first triangle, *bca*, *ad* is the altitude or height of the triangle. In the triangle on the right, *cd* is the altitude.

Tell what effect in each triangle the altitude has on the base and on the whole isosceles triangle.

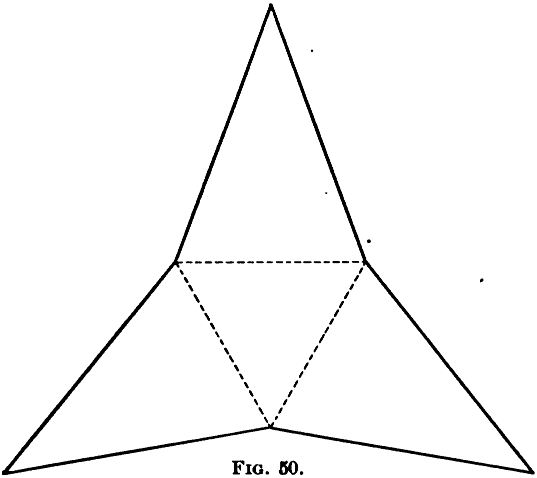


Fig. 50.

2. Study Figures 50 and 51 and, with their help, construct two *three-sided* or *triangular pyramids*.

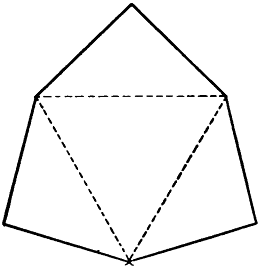


Fig. 51.

The equilateral triangle on which the pyramid stands is its *base;* the isosceles triangles are its *sides;* and the highest point in which the three sides meet is its *vertex.*

3. On a square base, construct a *four-sided* or *square pyramid* (Fig. 52).

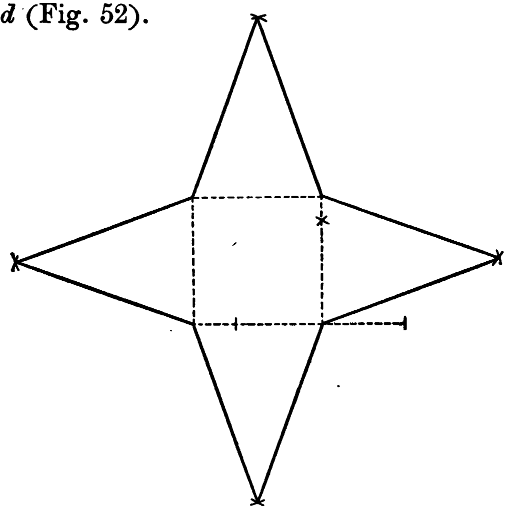


FIG. 52.

4. On a hexagonal base, construct a *six-sided* or *hexagonal pyramid.*

PROBLEM 16. — *How to draw a pentagon on a given side* (Fig. 53).

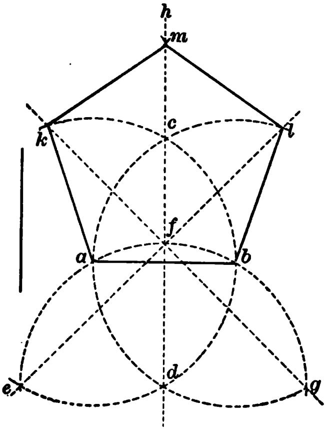


FIG. 53.

Draw the line *ab* of given length. With *ab* as radius and *a* and *b* successively as centres, draw arcs intersecting at *c* and *d*. With the same radius and *d* as centre, draw the arc *efg*. Through the point *f* draw the straight lines *dh*, *gk*, and *el*. With *ab* again as radius and *l* as centre, find *m*. Connect *b*, *l*, *m*, *k*, *a*. *ablmk* is the required *pentagon*, or five-sided figure.

EXERCISES.

1. Draw a pentagon as in Figure 53; and then in the pentagon connect *a* with *m*, *m* with *b*, *b* with *k*, *k* with *l*, *l* with *a*. You will have inscribed in the pentagon a *pentagonal* or *five-rayed star*.
2. In a pentagon draw a design, using only straight lines, and color it to your taste.
3. In a pentagonal star draw a design, using only straight lines, and color it to your taste.
4. Construct a *pentagonal* or *five-sided pyramid*.

PROBLEM 17. — *How to draw a pentagon equal to another (given) pentagon.*

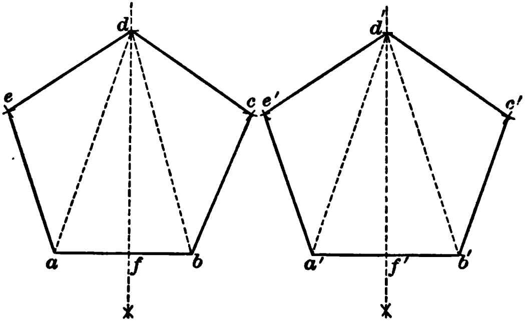


FIG. 54.

Study Figure 54, in which the problem is solved. Explain the construction and repeat it. (*abcde* is the given pentagon, constructed as in Figure 53. *a'b'c'd'e'* is copied from it with the help of the three dotted lines.)

EXERCISES.

1. Draw a pentagon, and on each side of it construct another pentagon equal to the first.
2. Construct a *pentagonal* or *five-sided prism.*

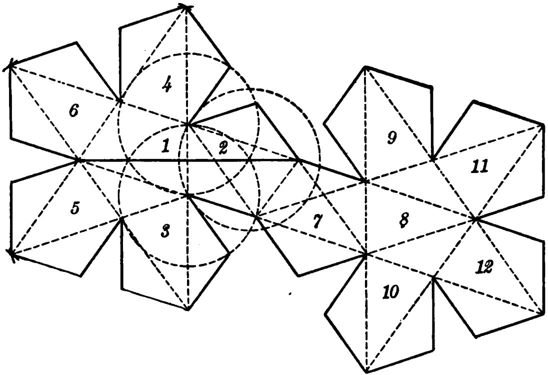


FIG. 55.

3. Study Figure 55, draw a similar one on cardboard or stiff paper; fold the pentagons inward on the dotted lines in which the pentagons touch; and construct the *dodecahedron*, or *twelve-sided solid.*

PROBLEM 18.—*How to inscribe a pentagon in a circle* (Fig. 56).

Draw the circumference of a circle and the two perpendicular diameters *ab* and *cd*. With the radius of the circle, and *b* as centre, draw the arc *eof*. Draw *egf*, or merely mark the point *g* in the direction of *egf*. With *g* as centre, and the distance *gd* as radius, draw the arc *dh*. Then the chord *dh* is the length of the side of the pentagon. With this chord as radius, mark the points *i*, *k*, *l*, *m* in the circumference, and draw the chords *di*, *ik*, *kl*, *lm*, *md*, the perimeter of the inscribed regular pentagon.

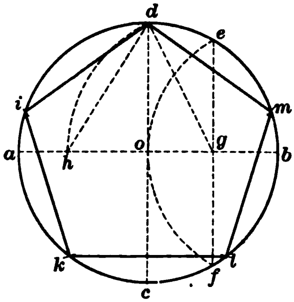


FIG. 56.

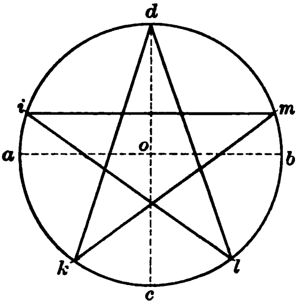


FIG. 57.

EXERCISES.

1. Inscribe a pentagon in a circle, draw a design in the pentagon, and color it to your taste.

2. Inscribe a pentagonal star in a circle (Fig. 57).

(The points in Figures 57 and 58 are lettered the same as corresponding points in Figure 56.)

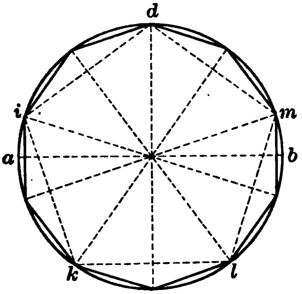


FIG. 58.

3. Inscribe a regular *decagon*, or *ten-sided figure*, in a circle (Fig. 58).

Problem 19.—*How to inscribe a heptagon in a circle* (Fig. 59).

Draw the circumference of a circle and its diameter, *ab*. With the same radius and *b* as centre, draw the arc *cod*. Draw the chord *cd*, and indicate the point *e*. Then *ce* is the length of the side of the inscribed *heptagon*, or seven-sided figure. With *ce* as radius mark in the circumference of the circle the points *f*, *g*, *h*, *i*, *k*, *l*, and draw *bf*, *fg*, *hi*, *ik*, *kl*, *lb*, the perimeter of the inscribed heptagon.

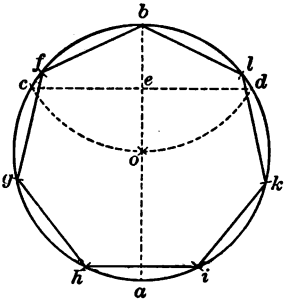


Fig. 59.

EXERCISES.

1. Inscribe a seven-rayed star in a circle.
2. Draw a design on the basis of an inscribed heptagon, using only straight lines.
3. Draw a design on the basis of an inscribed heptagon, using only curved lines.
4. Draw a design on the basis of an inscribed heptagonal star.

Problem 20. — *How to draw an angle equal to a given angle* (Fig. 60).

Draw any angle, *acb*. Draw a straight line, *c′b′*. With any radius and *c* as centre, draw an arc across the sides of the given angle, *acb*. With the same radius and *c′* as centre, draw a similar arc across the line *c′b′* on the side on which you wish to draw the new angle. Measure *ed* and make *e′d′* equal to *ed*. Through *d′* draw *c′a′*. Then the angle *a′c′b′* will be equal to the angle *acb*.

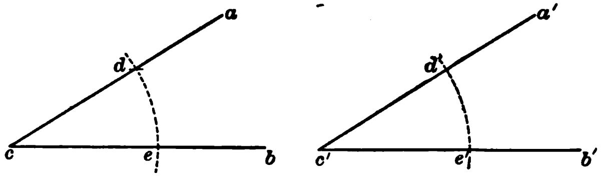


Fig. 60.

EXERCISES.

1. Draw Figure 16. Compare by measurement the arcs over each of the six angles. With a smaller radius and the centre *o*, draw a circumference within the circle. Compare by measurement the new arcs between the sides of the six angles. How many degrees in each angle?

2. Draw an equilateral triangle, *abc* (Fig. 41). With the radius with which you drew the inner circumference in the last Exercise and *a*, *b*, *c*, successively as centres draw arcs between the sides of the angles. Compare these arcs by measurement with each other, and also with the inner arcs between the sides of the six angles at the centre

of the circle in Exercise 1. How many degrees in each angle of the equilateral triangle?

3. The angle *bac* in Figure 61 measures 60°. How was it drawn? Draw an angle of 60 degrees in the same way.

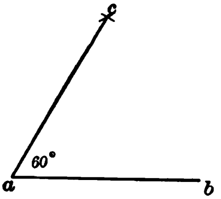


FIG. 61.

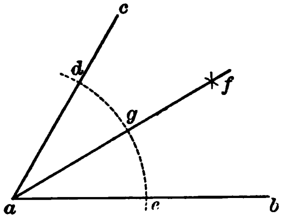


FIG. 62.

4. In Figure 62, the angle *bac* is an angle of 60°, *af* bisects it, *i.e.* divides it into two equal angles, each of 30°. Explain the process and repeat the construction.

5. Divide an angle of 60° into four equal angles of 15° each.

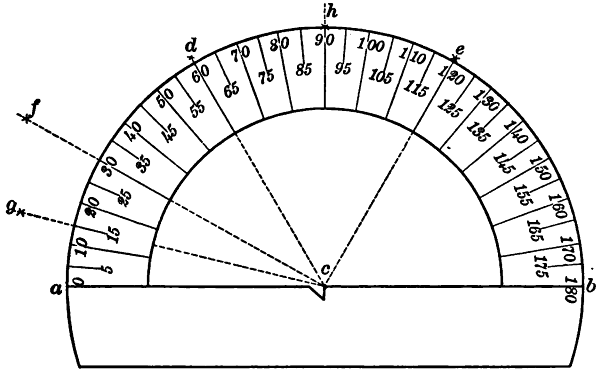


FIG. 63.

6. Figure 63 represents a *protractor*, an instrument for measuring arcs and angles. See if you can explain its

construction. The dotted lines are meant merely to help you in your study. You can make a serviceable protractor out of cardboard or celluloid.

7. Draw the following triangles:

Base 2 in., angle on left 30°, angle on right 30°.
Base 2 in., angle on left 75°, angle on right 75°.
Base 3 in., angle on left 45°, angle on right 45°.
Base 3 in., angle on left 75°, angle on right 45°.

PROBLEM 21.—*How to draw a line parallel to a given line* (Fig. 64).

The line *ab* is given; a line parallel to *ab* is to be drawn through the point *c*. Select any point, *d*, in the given line, *ab*. Draw a straight line from *d* through *c*. With *dc* as radius and *d* as centre, draw the arc *ce*; and with the same radius and *c* as centre, draw a similar arc on the same side of *dcf*. Measure *ce*, and mark off *fg* equal to *ce*. Through *c* and *g* draw the line *mo*. *mo* is parallel to *ab*; *i.e.* it runs in the same direction.

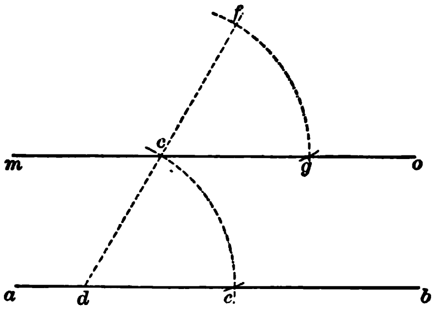


FIG. 64.

EXERCISES.

1. How does the angle *fcg* (Fig. 64) compare with the angle *cde*? Can you find another angle equal to *fcg*? If so, prove it by construction. Which angle, if any, is equal to the angle *mcf*? Prove your answer by construction.

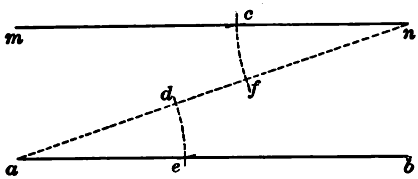


FIG. 65.

2. Figure 65 teaches another method of drawing through the point *c* a line parallel to the given line *ab*. Explain the method and repeat the construction.

3. In Figure 66, the opposite sides *ab* and *cd*, *ac* and *bd*, are respectively equal and parallel. Such a figure is called a *parallelogram*. Other parallelograms that you have drawn are the *square*, the *oblong rectangle*, and the *rhombus*

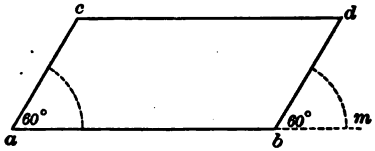


FIG. 66.

or *lozenge*. The parallelogram in Figure 67 is called a *rhomboid*. How does it differ from the rhombus? from the oblong rectangle? How many degrees in each of the obtuse angles?

Tell how the rhomboid was drawn; construct one similar to it; and show by construction that the opposite angles are equal.

4. Draw a group of four rhomboids to please your fancy, making the acute angles 45°, the longer sides 2 inch, the shorter sides 1½ inch.

PROBLEM 22. — *How to divide a given straight line into any number of equal parts* (Fig. 67).

The line *ab*, drawn of any length, is to be divided into three equal parts. From the point *a*, draw *ae* at any acute angle and of any length. From *b*, draw *be′* at the same angle and of about the same length. On the line *ae*, mark three equal distances, — *ac*, *cd*, *de*; and from *b* on the line *be′*, three equal distances of the same length, — *bc′*, *c′d′*, *d′e′*. Connect *eb*, *dc′*, *cd′*, and *ae′*. Then *dc′* and *cd′* will cut the line *ab* so as to divide it into three equal parts. Which of the connecting lines may be omitted in this construction?

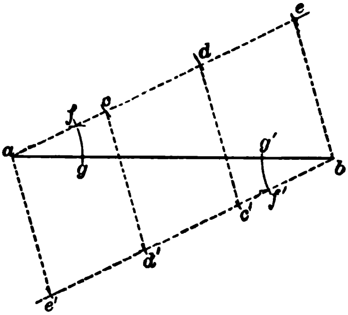


FIG. 67.

EXERCISES.

1. Divide a line of four inches into five equal parts.

2. Draw a rhombus; divide its longer diagonal into five equal parts, and connect the points of division with the vertices of the acute angles. (In making the division, it is best to draw a line equal to the longer diameter, to divide this into five equal parts, and then to transfer the divisions to the longer diagonal of the rhombus.)

3. Divide a square into nine equal smaller squares.

4. Draw an equilateral triangle four inches to the side; divide each side into seven equal parts; draw a design in the triangle on the basis of this division.

PART II.

COMPOSITE CURVES.

Problem 23. — *How to draw wavy lines.*

a. Figure 68 shows the construction of a wavy line, consisting of semicircles drawn alternately on opposite sides of a straight line.

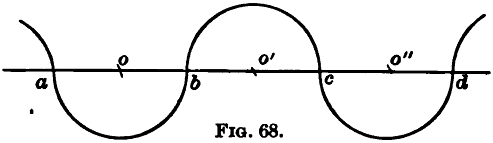


Fig. 68.

EXERCISES.

1. Draw a similar wavy line with a radius of ¼ inch.
2. Draw a similar wavy line with a radius of ¼ inch around an equilateral triangle, 3½ inches to the side.

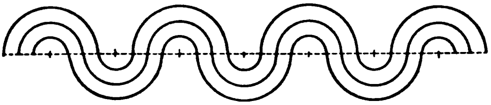


Fig. 69.

3. Draw three parallel wavy lines, as in Figure 69, making the radius of the middle curve ¼ inch, the radius of the larger semicircles ⅜ of an inch, the radius of the smaller semicircle ⅛ inch.

b. Figure 70 shows the construction of another wavy line. The three straight lines are parallel at distances of $\frac{1}{2}$ inch. The radius of the curves is $\frac{5}{8}$ of an inch; o^1, o^2, etc., are the centres of the curves.

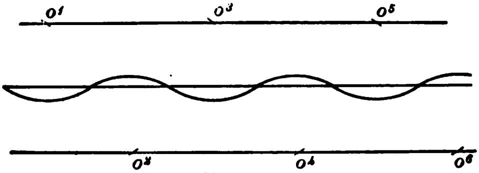


FIG. 70.

EXERCISES.

1. Draw a similar wavy line.
2. Draw a similar wavy line around a square.
3. Draw two parallel wavy lines of similar character.

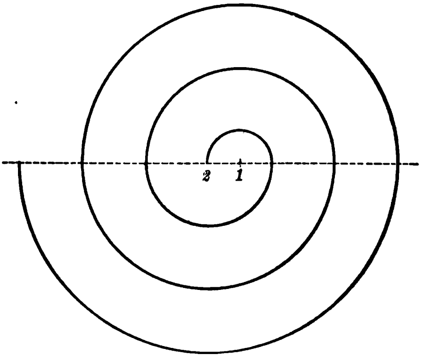


FIG. 71.

PROBLEM 24. — *How to draw a watch-spring spiral.*[1]

a. Figure 71 shows the construction. Notice that the semi-circumferences are drawn from two centres (*1* and *2*), and that on each side of the straight line the semi-circumferences are parallel.

---

[1] This and the following spirals are only approximations to true mathematical spirals.

b. Another method is indicated in Figure 72. Here four centres are used for the spiral line. Notice that the diameter of the small circle in the middle of the figure is divided into four equal parts; that the small square is drawn on two of these parts; that the dotted lines are prolongations of the sides of the small square; and that the quadrants drawn from the centres *1, 2, 3, 4,* are respectively parallel.

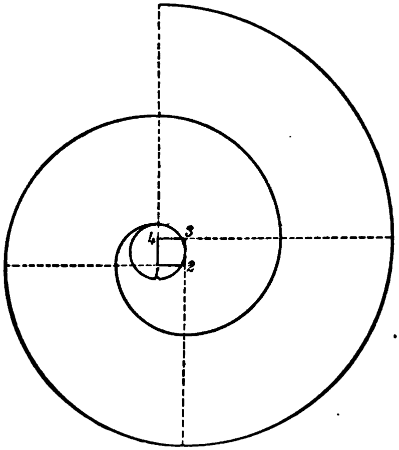


Fig. 72.

EXERCISES.

1. Draw two watch-spring spirals winding in opposite directions.

2. Draw two parallel watch-spring spirals winding around the same centre.

Problem 25.—*How to draw a diverging spiral* (Fig. 73).

Draw a straight line. With a short radius ($\frac{1}{8}$ inch) and point *1* as centre, draw the central circumference.

With double the radius and point *2* as centre, draw the semi-circumference *34*. Again doubling the radius and with point *3* as centre, draw *45*; etc.

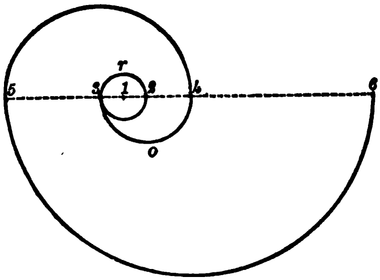


Fig. 73.

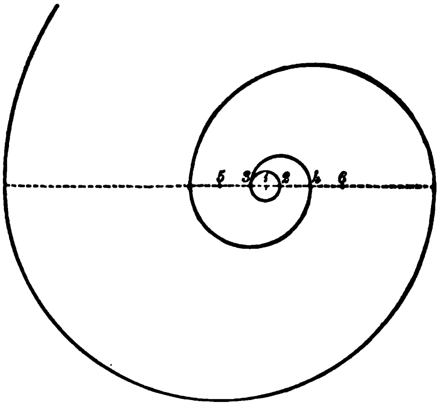


Fig. 74.

Figure 74 shows a similar spiral, drawn from the successive centres *1, 2, 3, 4, 5, 6*. The first radius is $\frac{1}{8}$ inch; the third, $\frac{3}{8}$ inch; etc.

EXERCISES.

1. Draw spirals similar to Figures 73 and 74.
2. Draw diverging spirals winding in opposite directions, making the distances between successive centres $\frac{1}{16}$ inch.

Problem 26. — *How to draw elliptical curves.*

An elliptical curve (Fig. 75) has two diameters differing in length. The longer diameter, *ab*, is called the *major axis;* the shorter diameter, *ed*, is called the *minor axis.* The points *f* and *g* are the *foci* of the ellipse. Their distance from the ends of the minor axis is equal to half the length of the major axis, *i.e.* for the focus *f*, *ef* and *df* are each equal to *ac*; and for the focus *g*, *eg* and *dg* are each equal to *ac*. If we take any point in the periphery of the ellipse, for instance the point *m*, the sum of its distances from the two focuses will be equal to the major axis, or $fm + gm = ab$. The same is true of every other point in the elliptical curve.

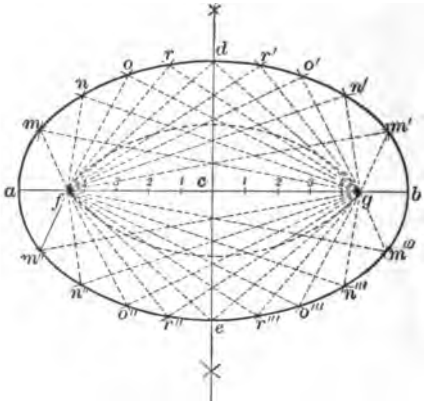


FIG. 75.

It is difficult to draw an exact elliptical curve. Therefore, easier ways to draw *approximate* elliptical curves have been invented.

a. ***How to draw an approximate elliptical curve of which the minor axis is given.*** — For Figure 76 the minor axis is given at $1\frac{1}{2}$ inches. Draw a straight line, *rs*, of indefinite length. At some point, *o*, near the middle of *rs*, draw a perpendicular to *rs*. Make *oc* and *od* respectively equal to one-half the given minor axis. Select any two points, *m* and *n*, equally distant from *o*. Draw *cmf*, *dme*, *dng*, and *cnh* of indefinite length. With *cd* as radius and *c* and *d* successively as centres, draw the arcs *fdh* and *ecg*. With *me* as radius and the points *m* and *n* successively as centres, draw the arcs *eaf* and *gbh*. *ab* will be the major axis of the approximate elliptical curve you have drawn.

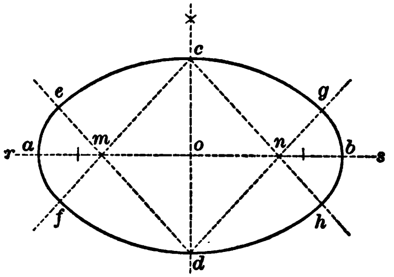


FIG. 76.

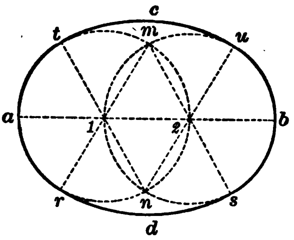


FIG. 77.

b. ***How to draw an approximate elliptical curve of which the major axis is given.*** — For Figure 77 the major axis is given at 2 inches. Divide the major axis *ab* into three equal parts *a1*, *12*, *2b*. With *a1* as radius and *1* and *2* successively as centres, draw two circumferences of circles, intersecting at *m* and *n*. Draw *m1r*, *n1t*, *m2s*, and *n2u*. With *nt* as radius and *n* and *m* successively as centres, draw the arcs *tcu* and *rds*. With *1a* as radius and *1* and *2* successively as centres, draw *rat* and *sbu*.

c. ***How to draw an approximate ellipse of which both axes are given.*** — For Figure 78 the major axis is given at $2\frac{1}{2}$ inches and the minor axis at 2 inches. Draw the major axis *ab*. Draw the minor axis *cd*, perpendicular to *ab* at its middle point *o*, making *oc* equal to *od*. With *oc* as radius and *o* as centre, draw the quadrant *ce*. Bisect *ae* at *m*. From the point *o* mark off on the major axis on both sides of the minor axis three divisions equal to *am*. Similarly mark off on the minor axis on both sides of the major axis four divisions *o, 1, 2, 3, 4.* Draw *43f*, *43g*, *43h*, and *43i* of indefinite length. With *4oc* as radius and the two points *4* successively as centres, draw the arcs *gch* and *fdi*. With *3g* as radius and the points *3* on the major axis successively as centres, draw the arcs *fag* and *hbi*.

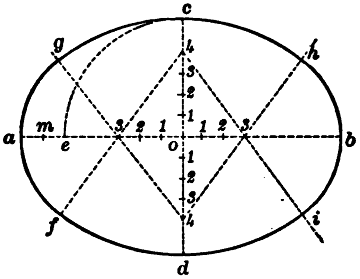


FIG. 78.

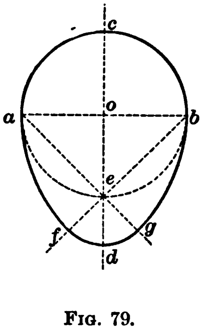


FIG. 79.

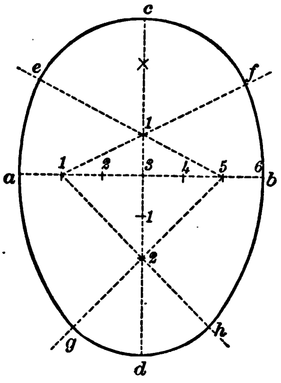


FIG. 80.

d. *How to draw a complex curve resembling the cross-section of a hen's egg.* — The problem is solved in Figure 79. Study it, explain it, and repeat the construction. Another method is shown in Figure 80. Study it and repeat it.

APPENDIX I.

DEFINITIONS OF GEOMETRICAL TERMS.

Magnitudes.

1. *Magnitude* means size or extent.

In geometry things that have size or extent are called *magnitudes*.

There are three kinds of magnitudes, — *lines*, *surfaces*, and *solids*.

The size or extent of a magnitude is determined by measuring.

2. The direction in which a magnitude is measured, in order to determine its extent, is called a *dimension*.

Magnitudes may have one or two or three dimensions.

3. A *line* is a magnitude of one dimension, — *length*.

4. A *surface* is a magnitude of two dimensions, — *length* and *breadth*. Squares and circles are surfaces.

5. A *solid* is a magnitude of three dimensions, — *length*, *breadth*, and *thickness*. Cubes and prisms are solids.

6. The extent of a line is its *length*.

7. The extent of a surface is its *area*.

8. The extent of a solid is its *bulk* or *volume*.

The length of a line is determined by *long measure*.

The area of a surface is determined by *square measure*.

The volume of a solid is determined by *cubic measure*.

9. A *point* is mere position. It has no size or extent.

Review:

Magnitudes.
- *Lines*—one dimension, length, long measure.
- *Surfaces*—two dimensions, area, square measure.
- *Solids*—three dimensions, volume, cubic measure.

Lines and Angles.

In character, lines are either *straight* or *curved.*

1. A *straight* line is one that has the same direction in its whole extent. The outline of a square is made up of four straight lines.

2. A *curved* line is one that changes its direction at every successive point. The outline of a circle is a curved line.

In relative direction, lines are *parallel* or *diverging.*

3. Lines running in the same relative direction are said to be *parallel.* The opposite sides of a square and the circumferences of circles that have the same centre are parallel.

Parallel lines are at the same distance from each other in all parts. They can never meet in a common point, no matter how much they may be prolonged.

4. Lines running in relatively different directions are said to be *diverging.* The adjacent sides of a square are diverging.

They are said to *diverge* in the direction in which they recede from each other, and to *converge* in the direction in which they approach each other.

When prolonged in the direction in which they converge, they will meet in a common point. On each side of this point they diverge.

5. Two diverging lines proceeding from a common point are said to form an *angle.* The *angle* is the amount

of divergence, or difference in direction, between the two lines. The greater the divergence, the greater the angle.

6. The two lines forming the angle are its *sides* or *arms*.

7. The point in which the two sides meet, or from which the two sides diverge, is the *vertex* of the angle.

If one line meets another line in any point between the ends of this other line, the two lines form two angles.

If two lines cross or intersect each other, they form four angles.

Diverging lines are either *perpendicular* or *oblique* to each other.

8. They are said to be *perpendicular* to each other, if, on being prolonged so as to intersect each other, they form four equal angles.

9. They are said to be *oblique* to each other, if, on being prolonged so as to intersect each other, they form unequal angles.

10. Angles formed by *perpendicular* lines are said to be *right* angles.

11. Angles formed by *oblique* lines are said to be *oblique* angles.

12. An oblique angle, smaller (or of less divergence) than a right angle, is said to be *acute*.

13. An oblique angle, larger (or of more divergence) than a right angle, is said to be *obtuse*.

Review:

- **Lines**
  - in character
    - straight
    - curved
  - in relative direction
    - parallel
    - diverging
      - perpendicular
      - oblique
- **Angles**
  - right
  - oblique
    - acute
    - obtuse

14. A line, parallel to the plumb-line, is said to be *vertical.*

15. A line, parallel to the horizon, is said to be *horizontal.*

Vertical and horizontal lines are perpendicular to each other.

16. Lines, oblique to the horizon and to the plumb-line, are said to be *slanting.*

Review:

Lines with reference to the horizon and the plumb-line	vertical horizontal slanting

Surfaces.

Surfaces are either *plane* or *curved.*

1. Surfaces are said to be *plane*, if both dimensions (length and breadth) are straight lines. The square and the circle are plane surfaces.

2. Surfaces are said to be *curved*, if one or both dimensions be curved. The side of a cylinder has one straight and one curved dimension. The surface of a sphere has both dimensions curved.

3. A plane figure bounded by a curved line is said to be *curvilinear.*

4. A plane figure bounded by straight lines is said to be *rectilinear.*

5. A rectilinear plane figure is also called a *polygon*, *i.e.* a figure of many angles.

The principal curvilinear planes are the *circle*, the *ellipse*, and the *oval.* The principal rectilinear planes are the *triangle*, the *quadrilateral* (four-sided figure), the *pentagon* (five-sided), *hexagon* (six-sided), *heptagon* (seven-sided), *octagon* (eight-sided), *decagon* (ten-sided), and *dodecagon* (twelve-sided figure).

Review:

- **Surfaces**
  - curved
    - cylindrical
    - spherical
  - plane
    - curvilinear
      - circle
      - ellipse
      - oval
    - rectilinear
      - triangle
      - quadrilateral
      - pentagon
      - hexagon
      - etc.

The Circle.

1. A *circle* is a curvilinear plane figure bounded by a single uniformly curved line. All points of this line are equally distant from a point within the circle called its *centre*.

2. The line bounding a circle is its *circumference* or *periphery*.

3. The *centre* of a circle is a point within the circle equally distant from all points of the circumference.

4. A straight line drawn from the centre to the circumference is a *radius* of the circle.

5. The *diameter* of a circle is a straight line drawn from any point in its circumference, through its centre, to the opposite point in its circumference.

6. Any diameter divides the circle into two equal parts, or *semicircles*. It also divides the circumference into two equal parts, or *semi-circumferences*.

7. Any straight line connecting two points in the circumference is called a *chord*. The diameter is the longest chord that can be drawn in a circle.

8. Any portion of a circumference of a circle is called an *arc*.

9. A portion of a circle cut off by a chord is called a *segment*. Any chord divides a circle into two segments.

10. Any portion of a circle included between two radii is called a *sector*.

11. A straight line that touches the circumference of a circle in only one point is called a *tangent*.

12. Different circles that have the same centre are said to be *concentric*.

Polygons.

1. A *polygon* is a rectilinear plane figure bounded by three or more lines (*poly* = many; *gon* = angle).

2. These lines are the *sides* of the polygon.

3. The sum of all the sides of a polygon is its *perimeter*.

4. In a *regular polygon*, the sides and the angles at the perimeter are respectively equal.

5. Within a regular polygon there is a point equally distant from all the vertices of the angles at the perimeter. This point is the *centre* of the regular polygon.

The centre of a regular polygon is also equally distant from the middle points of its sides.

6. Polygons are named from the number of their sides and angles. The most important polygons are the following:

a. Three-sided polygon, Triangle (*tri* = three).

b. Four-sided polygon, Quadrilateral (*quadrilateral* = four-sided).

c. Five-sided polygon, Pentagon (*penta* = five).

d. Six-sided polygon, Hexagon (*hexa* = six).

e. Seven-sided polygon, Heptagon (*hepta* = seven).

f. Eight-sided polygon, Octagon (*octa* = eight).

g. Ten-sided polygon, Decagon (*deca* = ten).

h. Twelve-sided polygon, Dodecagon (*dodeca* = twelve).

Triangles.

1. A *triangle* is a three-sided polygon.

2. A *regular* triangle which has all its sides and angles respectively equal is also called an *equilateral triangle* (*equilateral* = equal-sided).

3. A triangle which has two of its sides equal is said to be *isosceles* (*isosceles* = having equal legs).

4. A triangle which has all its sides of different lengths is said to be *scalene* (*scalene* = having crooked legs).

5. A triangle which has one of its angles a right angle is said to be a *right-angled* or a *right* triangle.

6. A triangle which has one of its angles an obtuse angle is said to be an *obtuse-angled* triangle.

7. A triangle which has all of its angles acute is said to be an *acute-angled* triangle.

Review:

- **Triangles**
  - named by their angles
    - right-angled
    - obtuse-angled
    - acute-angled
  - named by their sides
    - equilateral
    - isosceles
    - scalene

8. In a right triangle, the side opposite the right angle is called the *hypotenuse*. The other two sides are the *sides* forming the right angle.

9. A triangle may be supposed *to stand* on any one of its sides. This side is called the *base* of the triangle. Any one of the sides of the triangle may be considered as its base. In an isosceles triangle, it is customary to consider the unequal side as the base.

10. The vertex of the angle opposite the base is called the *vertex* (top) of the triangle.

11. A line drawn from the vertex of a triangle perpendicular to the base or to its prolongation is the *altitude* (height) of the triangle.

In each of the following triangles, the base is marked b, the vertex v, and the altitude a.

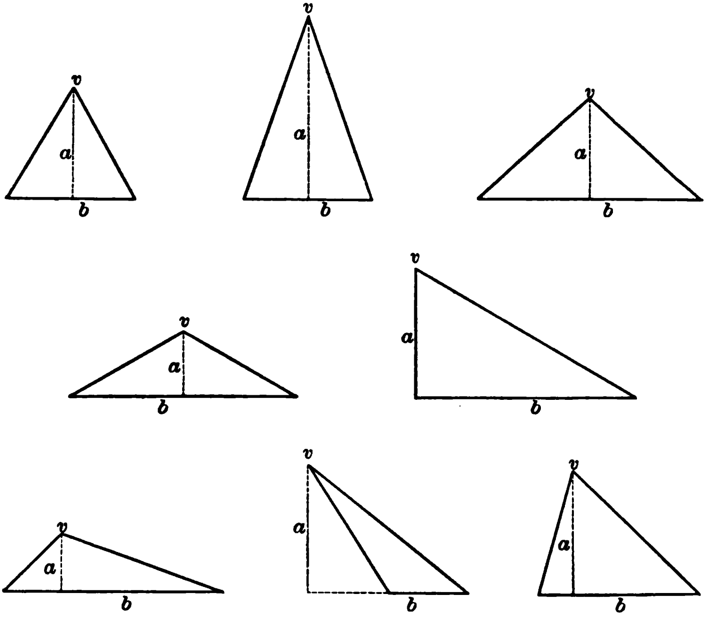


Fig. 81.

Quadrilaterals.

1. A *quadrilateral* is a four-sided polygon.

2. A *parallelogram* is a quadrilateral which has the two pairs of opposite sides respectively equal and parallel.

3. A *trapezium* is a quadrilateral which has no two sides parallel.

4. A *trapezoid* is a quadrilateral which has two of its opposite sides parallel.

5. A parallelogram whose angles are right angles is called a *right parallelogram* or *rectangle.*

6. A parallelogram whose angles are oblique is an *oblique parallelogram.*

7. A *square* is a right parallelogram. All its sides are equal; it may, therefore, be described as an *equilateral right parallelogram.*

Because it is a polygon whose sides and angles are respectively equal, it may also be described as a *regular quadrilateral.*

8. An *oblong rectangle* is a right parallelogram whose adjacent sides differ in length.

9. A *rhombus* or *lozenge* is an equilateral oblique parallelogram.

10. A *rhomboid* is an oblique parallelogram whose adjacent sides differ in length.

11. A straight line which connects the vertices of the opposite angles of a quadrilateral is called a *diagonal* (*dia* = through).

12. Any side of a quadrilateral may be considered as its *base*, and a line drawn from the vertex of one of the opposite angles perpendicular to the base is its *altitude.*

In the following quadrilaterals, diagonals are indicated in dotted lines.

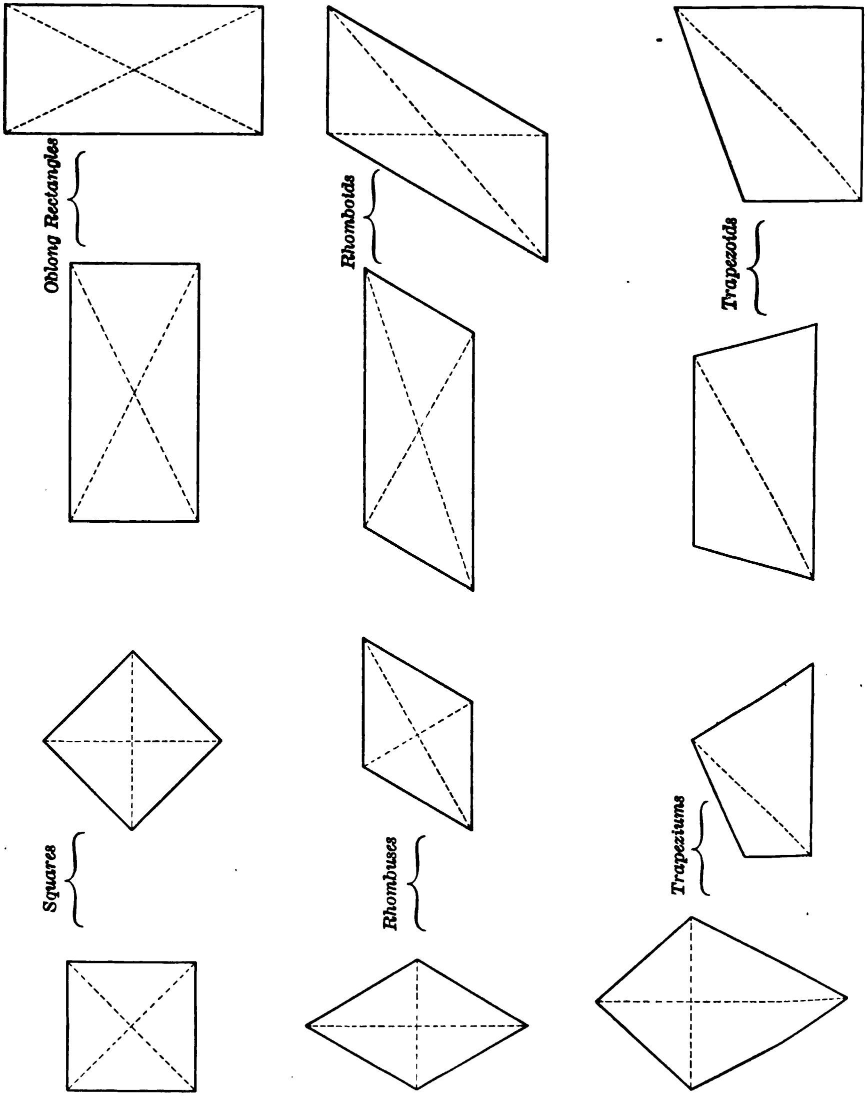

Oblong Rectangles
Rhomboids
Trapezoids
Squares
Rhombuses
Trapeziums

FIG. 82.

The Ellipse. (See Figure 75.)

1. The ellipse is a curvilinear plane which has in its longest diameter two fixed points, the sum of whose distances from any point in the circumference is equal to this longest diameter.

Thus,

$$fo + go = ab$$

$$fm + gm = ab$$

$$fa + ga = ab, \text{ etc.}$$

2. The longest diameter, *ab*, or the *length* of the ellipse is also called its *major* (larger) *axis*.

3. The shortest diameter, *de*, or the *breadth* of the ellipse is also called its *minor* (smaller) *axis*.

4. The two points in the major axis, jointly equally distant from every point in the circumference are the *foci* or *focuses* of the ellipse.

The distance of each focus from the ends of the minor axis is equal to one-half the major axis.

Polyhedrons.

1. A *polyhedron* is a solid bounded by polygons.

2. Each of the polygons in the surface of a polyhedron is called a *face*.

3. A *regular polyhedron* is one whose surface is made up of equal, regular polygons.

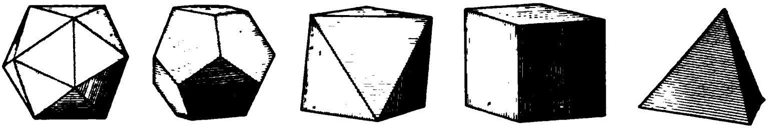
4. A regular polyhedron of four faces (equilateral triangles) is a *tetrahedron* (page 23).

5. A regular polyhedron of six faces (squares) is a *hexahedron* or *cube* (page 17).

6. A regular polyhedron of eight faces (equilateral triangles) is an *octahedron* (page 24).

7. A regular polyhedron of twelve faces (pentagons) is a *dodecahedron* (page 31).

8. A regular polyhedron of twenty faces (equilateral triangles) is an *icosahedron* (page 25).



Icosahedron *Dodecahedron* *Octahedron* *Hexahedron (Cube)* *Tetrahedron*

FIG. 83.

9. A *prism* is a polyhedron whose end faces or bases are polygons, having their corresponding sides parallel, and whose side faces are parallelograms.

10. A *right prism* is one whose side faces are perpendicular to the bases.

11. An *oblique prism* is one whose side faces are oblique to the bases.

12. A *regular prism* is a right prism whose bases are regular polygons. Their side faces are rectangles.

13. Prisms are named from the shape of their bases: *triangular* prism, *square* prism, *pentagonal* prism, etc. (see pages 18, 24, etc.).

14. A *pyramid* is a polyhedron which has a polygon for its base, and as many triangular side faces meeting in a common point as the base has sides.

15. The common point in which the side faces meet is the *vertex* of the pyramid.

16. A *regular pyramid* is one whose base is a regular polygon and whose side faces are equal.

17. The *slant height* of a regular pyramid is the altitude of any one of its side faces.

18. Pyramids are named from the shape of their bases: *triangular*, *square*, *pentagonal*, etc., pyramids (see pages 28, 29, etc.).

Other Solids.

1. A *sphere* is a solid bounded by a single uniformly curved surface. All points of this surface are equally distant from a point within the sphere called its *centre.*

2. The *centre* of a sphere is a point within the sphere equally distant from all points of the surface.

3. The *radius* of a sphere is a straight line drawn from its centre to its surface.

4. The *diameter* of a sphere is a straight line passing through its centre and connecting opposite points of the surface of the sphere.

5. A circle whose centre is at the centre of the sphere and whose circumference lies in the surface of the sphere is called a *great circle.* Its radius is the same as the radius of the sphere.

6. The diameter of a sphere, perpendicular to a great circle, is the *axis* of that circle.

7. The ends of the axis are the *poles* of that circle.

8. Any great circle bisects the sphere into two *hemi*spheres.

9. A *lune* is a portion of a spherical surface included between two semi-circumferences of great circles.

10. If a sphere is flattened at the poles of one of its great circles and the axis shortened, the sphere is changed into an *oblate spheroid.*

11. If a circle is stretched at the poles of one of its great circles and its axis prolonged, the sphere is changed into a *prolate spheroid.*

12. A *cylinder* is a solid bounded by two equal circular ends and a uniformly curved side.

13. The perpendicular distance between the two circular ends or bases is the *altitude* of the cylinder.

14. A *cone* is a solid bounded by a circular base and a uniformly curved side running into a point.

15. This point is the *apex* of the cone.

16. The distance between the apex and the circumference of the base is the *slant height* of the cone.

NOTE. — For the sphere, cylinder, and cone, no constructive exercises have been suggested. Should the teacher desire to enter this field, he will find in cardboard and plastic clay satisfactory material. For the construction in cardboard, modern text-books on geometry furnish sufficient hints. Many pupils will succeed, however, in discovering independently correct methods for the cardboard construction of the cylinder and cone, but not for the sphere and spheroids.